# A Planar Perspective Image Matching using Point Correspondences and Rectangle-to-Quadrilateral Mapping

Dong-Keun Kim Department of Computer and Information Science, Seonam University Jeonbuk, Namwon, Korea, <u>dgkim@tiger.seonam.ac.kr</u>

Byung-Tae Jang Electronics and Telecommunications Research Institute(ETRI) Taejeon, Korea, jbt@etri.re.kr

Chi-Jung Hwang Department of Computer Science, Chungnam National University Taejeon, Korea, <u>cjhwang@ipl.chungnam.ac.kr</u>

### ABSTRACT

In this paper, we considered a planar perspective transformation between images with overlapping region. It is based on the rectangle-to-quadrilateral mapping and normalized correlations. Initially, the global translation is determined by using a block matching. And to find the correspondence points maximizing correlation in overlapping region by the perspective transformation, we used simulated annealing (SA) algorithm based on the normalized correlation. In the block matching and SA, we used Gaussian pyramid structure. We show results of applying our proposed algorithm to mosaic images.

# 1. Introduction

Image mosaicing can create a large image from a sequence of images with overlapping region. It is the process of warping a sequential images captured from a real world scene, and generating a single large image. We can create the mosaic images by image alignment and blending, first is to warp the images by the transformation between overlapping images. Second is to blend images warped by the transformation. There are several transformations that we can use to warp images. That is pure translation, rigid, affine, and perspective transformation.

To warp images, we use the perspective transformation which has eight degrees of freedom. It can be determined

by minimizing the sum squared of the difference intensity between overlapping areas using Levenberg-Marquardt algorithm [1, 2, 3].

In this paper, we considered finding the perspective transform between the overlapping region by the normalized correlation and rectangle-to-quadrilateral mapping. The perspective transformation has eight degrees of freedom. So the eight coefficients can be determined by using four correspondence points between the reference and target images. The eights coefficients can determined by using rectangle-to-quadrilateral mapping.

In the next section, we briefly review of a rectangle-toquadrilateral mapping, and propose our methods in the next section, the following section shows experimental results to merge three images.

# 2. Planar Perspective Transformation

A general planar perspective transformation can be represented as:

$$M = \begin{bmatrix} m_0 & m_3 & m_6 \\ m_1 & m_4 & m_7 \\ m_2 & m_5 & 1 \end{bmatrix}$$



For the perspective transformation M, the forward transformations are

$$x' = \frac{m_0 x + m_1 y + m_2}{m_6 x + m_7 y + 1}$$
(1)  
$$y' = \frac{m_3 x + m_4 y + m_5}{m_6 x + m_7 y + 1}$$

The perspective transformation has eight degrees of freedom. So the eight coefficients can be determined by using four correspondence points between the reference I(x,y) and target images I(x', y'). The eight coefficients are determined by solving the linear system. It is possible to speed up in case of a rectangle-to-quadrilateral mapping [5, 6]. We briefly review of a rectangle-to-quadrilateral mapping. We will consider mapping a rectangle to an arbitrary quadrilateral like figure 1. The four correspondence points are (x<sub>0</sub>, y<sub>0</sub>), (x<sub>1</sub>, y<sub>1</sub>), (x<sub>2</sub>, y<sub>2</sub>), (x<sub>3</sub>, y<sub>3</sub>) on reference image and (x<sub>0</sub>', y<sub>0</sub>'), (x<sub>1</sub>', y<sub>1</sub>'), (x<sub>2</sub>', y<sub>2</sub>'), (x<sub>3</sub>', y<sub>3</sub>') on target image.



Figure 1. Rectangle-to-quadrilateral mapping

### 2.1 Unit Square-to- Quadrilateral Mapping

We first consider a unit square-to- quadrilateral mapping [5], i.e., mapping between (0, 0), (1, 0), (0, 1), (1, 1) and  $(x_0', y_0')$ ,  $(x_1', y_1')$ ,  $(x_2', y_2')$ ,  $(x_3', y_3')$ . The transformation is as:

$$A = \begin{bmatrix} a_0 & a_3 & a_6 \\ a_1 & a_4 & a_7 \\ a_2 & a_5 & 1 \end{bmatrix}$$

Where

$$\Delta x_{1} = x_{1} - x_{2}, \quad \Delta x_{2} = x_{3} - x_{2}$$

$$\Delta x_{3} = x_{0} - x_{1} + x_{2} - x_{3}$$

$$\Delta y_{1} = y_{1} - y_{2}, \quad \Delta y_{2} = y_{3} - y_{2}$$

$$\Delta y_{3} = y_{0} - y_{1} + y_{2} - y_{3}$$
(2)

$$a_{6} = \frac{\Delta x'_{3} \Delta y'_{3} - \Delta x'_{2} \Delta y'_{3}}{\Delta x'_{1} \Delta y'_{2} - \Delta y'_{1} \Delta x'_{2}}$$

$$a_{7} = \frac{\Delta x'_{1} \Delta y'_{3} - \Delta y'_{1} \Delta x'_{3}}{\Delta x'_{1} \Delta y'_{2} - \Delta y'_{1} \Delta x'_{2}}$$

$$a_{0} = x'_{1} - x'_{0} + a_{6} x'_{1}$$

$$a_{1} = x'_{3} - x'_{0} + a_{7} x'_{3}$$

$$a_{2} = x'_{0}$$

$$a_{3} = y'_{1} - y'_{0} + a_{6} y'_{1}$$

$$a_{4} = y'_{3} - y'_{0} + a_{7} y'_{3}$$

$$a_{5} = y'_{0}$$
(3)

### 2.2 Rectangle-to-Quadrilateral Mapping

Next we will consider a rectangle-to-quadrilateral mapping between  $(x_0, y_0)$ ,  $(x_1, y_1)$ ,  $(x_2, y_2)$ ,  $(x_3, y_3)$  on reference image and  $(x_0', y_0')$ ,  $(x_1', y_1')$ ,  $(x_2', y_2')$ ,  $(x_3', y_3')$  on target image. It can be accomplished by scale and translation of a unit square-to-quadrilateral mapping as:

$$\begin{bmatrix} u & v & w \end{bmatrix} = \begin{bmatrix} x & y & 1 \end{bmatrix} M \quad (4)$$
  
where  $M = T(-x_0, -y_0) S(\frac{1}{x_1 - x_0}, \frac{1}{y_2 - y_0}) A$ 

And also it can be find the inverse projective transform in quadrilateral-to-rectangle mapping as:

$$\begin{bmatrix} u & v & w \end{bmatrix} = \begin{bmatrix} x' & y' & 1 \end{bmatrix} M^{-1} \quad (5)$$
  
where  $M^{-1} = A^{-1} S(x_1 - x_0, y_2 - y_0) T(x_0, y_0)$ 



### 3. Finding the correspondence points

#### 3.1 Global Translations by Initial Block Matching

To find an initial transformation, we use a block matching by the normalized correlation. For each points  $(x_i, y_i)$  of the rectangle grid (e.g., 3×3) in reference image, we find the point,  $(x'_i, y'_i)$  of maximizing the normalized correlation using the restricted window (e.g., 7×7, 11×11). We have tested the matching points because a mismatch may be found.

For each correspondence point pairs  $(x_i, y_i)$  and  $(x'_i, y'_i)$ , we can calculate a translation between measure between the reference image I (x, y) and target I'(x', y'). Then we choose the translation of maximizing the normalized correlation equation (6) in the overlapping area by the translation. To guarantee a good initial solution, it can be used more grid points than four in reference image.

#### 3.2 Objective Function

We have considered the normalized correlation of intensity as similarity measure in overlapping area of the reference image I(x, y) and target I'(x', y'). It is defined within the overlapping region by the perspective transform of a rectangle-to-quadrilateral mapping. Inferring the planner perspective transform between two images is to find the eight coefficients of maximizing equation (6).

$$R_{1}(M) = \frac{E[I(x, y)I'(x', y')] - E[I(x, y)]E[I'(x', y')]}{\sigma_{I(x, y)}\sigma_{I'(x', y')}}$$
(6)

We assume that the right of the reference image is overlapping in the left of the target. And we fixed a rectangle in the reference image I(x, y).

#### 3.3 Finding the correspondence points by SA

The global translation by block matching is used as an initial solution for the perspective transform. To find the solution of maximizing the objective function (6), we use simulated annealing (SA) algorithm [7].

(1) Initialize temperature T, calculate the perspective transformation using

correspondence pairs by equations (2), (3), (4) and (5).

- (2) Determine the overlapping region between two images by the perspective transformation, and calculate the correlation, oldR by equation (6).
- (3) Perturb the points (x<sub>i</sub>', y<sub>i</sub>'), i=0,.., 3 in small range of the target image, and calculate the perspective transformation by equations (2), (3), (4) and (5), determine the overlapping region between two images, and calculate the correlation, newR by equation (6).
- (4) If oldR < newR then accept the perturbed points. Otherwise accept with probability p=exp[-(oldR-newR)/kT]. Iterate step (3) and (4) until |oldR newR| < a threshold.</li>

SA can converge in a few iterations because we can give an initial solution close to global. To find effectively the solution, we construct Gaussian pyramid structure of input images with four layers. First we perform our methods in the coarse layer. Then the previous coarse solution used in the next fine layer.

### 4. Experimental Results

In our experiments, we have considered the merging of three input images by using the proposed method. We considered the normalized correlation to find a global translation by block matching and to find the perspective transformation in overlapping area by SA. In the overlapping region, we blend two images by the linear weight function. Figure 2(a), 2(b) and 2(c) show results of finding the correspondence points from pairs of input images in my office. Figure 3 show results of applying our proposed algorithm to mosaic images. And also figure 3(a), 3(b) and 3(c) show results of finding the correspondence points from pairs of input images in my office. Figure 3(d) show results of applying our proposed algorithm to mosaic images.

Figure 2(d) and Figure 3(d) are results using both the inverse mapping between the left image (2(a), 3(a)) and the middle image (2(b), 3(b)), and the forward mapping between the middle (2(b), 3(b)) and the right image (2(c), 3(c)) respectively.





(b)

(a)

(c)



(d) Figure 2. Mosaic Image 1



(a)

(b)

(c)



(d) Figure 3. Mosaic Image 2



# 5. Conclusions

We have considered the perspective transformation between two images that are overlapping. It is based on a rectangle-to-quadrilateral mapping of four correspondence points. The four correspondence points are determined by maximizing the normalized correlations using a block matching and SA algorithm. We show some results of applying our proposed method to merge images.

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